Shock Wave (M-4) ①

→ A shock wave is a kind of pressure wave with a steep pressure rise

→ " A shock wave is a compression wave front in a supersonic flow field across which sudden changes of fluid properties occurs "

→ It is an instantaneous process which involves viscous dissipation and heat conduction of appreciable magnitude and hence is an irreversible process.

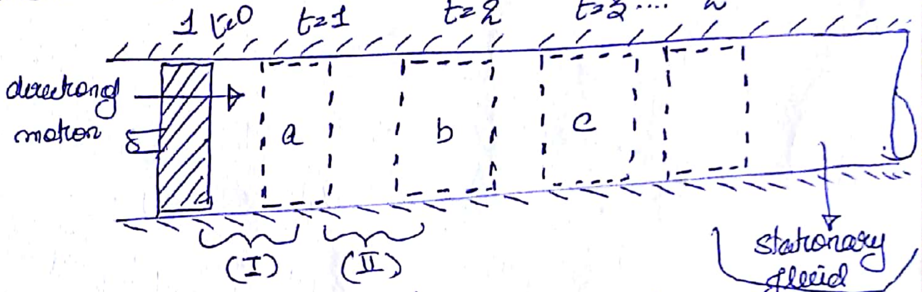
→ A shock wave results in sudden decrease in kinetic energy of fluid and flow turns subsonic after passing through it. A small part of kinetic energy is lost in viscous effects and sets in irreversibility which is reflect has increase in entropy. Hence a shock wave reduces the Available energy of fluid.

→ Any discontinuities occuring in a fluid flow cannot exist for a finite period of time because viscous effects, mass diffusions heat conduction tends to smooth these discontinuities. But shock waves are so thin that their discontinuity tends to remain in the flow field. The sudden change in kinetic energy (velocity) and pressure energy (pressure) occurs over a very short distance. It has been observed that the thickness of shockwave is of the order 10^{-7} mm.

Formation of a Shock Wave.

Consider a piston cylinder arrangement with a perfect gas in it. When the piston moves into the cylinder, it pushes the bulk of fluid inside it resulting in compression of fluid. The motion of piston sets up pressure waves (sound waves) which propagates into the fluid compressing it.

Let us consider that the fluid is initially in rest and the motion of piston is of ~~constant~~ gradual acceleration. Let this ~~actual~~ acceleration be approximated as an effect of series of instantaneous acceleration over a small equal time intervals. (i.e. acceleration from 1-2 occurs through a, b, c etc. ...)

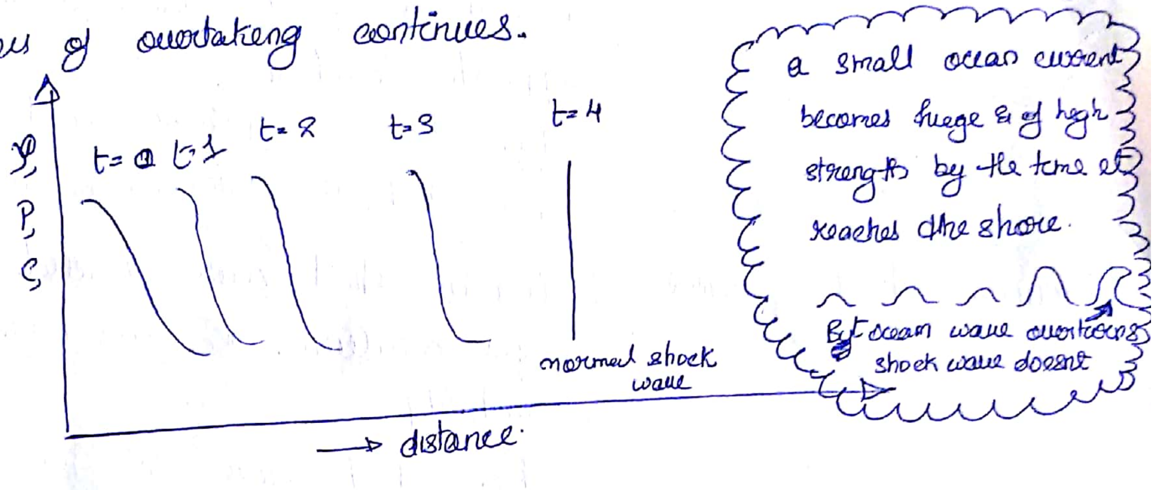


When the first impulsive (instantaneous) acceleration occurs from position '1' to 'a', a pressure wave has moved down the duct by a short distance and influenced the mass of gas in "I". Because of this this mass is at slightly increased pressure and temperature, and is moving to right with velocity of piston. Between 'a' & 'b' after time t=2, the mass receives further increase in pressure and velocity due to 2nd acceleration of piston. By this time the original wave would have travelled downstream influencing the mass and imparting a pressure and velocity. Thus at 'b' the mass (I & II) would have pressure due to piston motion and pressure waves (from 1 & a t=0 & t=1). Hence will be at an higher pressure value than that at 'a'. A new pressure wave ~~s~~ sets out and has moved down by the time piston is at 'b'.

Thus we see that at each new position a fresh pressure wave propagates downstream which are generated at condition of higher pressure and higher temperature than those produced in previous position.

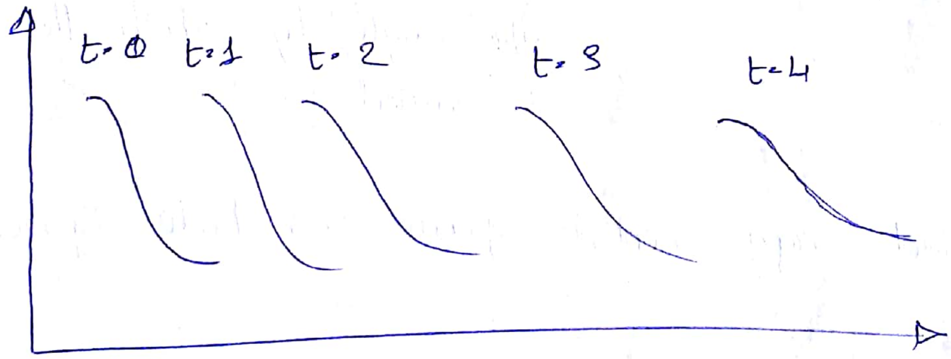
The velocity of pressure is that of velocity of sound. $(a=\sqrt{\gamma RT})$ and depends on temperature. Because of this the freshly generated pressure waves (which are at elevated pressure & temperature) travels with higher velocities and overtake the pressure waves produced before them.

The result of this is the waves becomes (3) stronger and more steeper under each others influence and thus a compression shock wave is formed. This moves into the fluid downstream and grows in strength as the process of overtaking continues.



A Compression wave can produce shock but rarefaction wave cannot.

→ Rarefaction waves are otherwise known as expansion waves. Consider the same process as in compression only difference piston is moving outward creating more space for the fluid cause it to expand. When fluid expands pressure and thereby the temperature drops. Hence successive sound waves generated at instantaneous times will have lower velocities ($\sqrt{\gamma RT}$) and they can never overtake the pressure waves before them. Hence they move into the bulk of fluid without strengthening and dies off.



at 'a' mass 'I' influenced by sound wave produced at t=0 & piston accl.

at 'b' mass 'I'+'II' influenced by sound waves produced at t=0 t=1 & piston accl.

⋮

so on.

Basic Classification

Stationary shock wave : When the gas in which shock wave propagates travel at speed equal to that of shock but in opposite direction.

Eg: shock in supersonic wind tunnel.

Eg: shock in diverging section of C-D nozzle

Moving shock wave : When shock wave travels in a stationary gas medium or gas moving with low velocity

Eg: shock wave set by explosion moving through air.

Normal shock wave : When shock wave is position at right angles to flow direction

1-D analysis is required _ simpler

Oblique shock wave :
* When shock wave is position at an angle to flow direction.

  2-D analysis is required.
* The inclination of shock wave is called wave angle (σ)
* The angle by which the flow gets deflected is denoted by 'δ'

If asked copy contents from gas table page 6 & 7.

Mach wave : Weak compression wave (low strength). Changes due to this wave is infinitesimal and hence can be approximated as isentropic.

This can be of compressive or expansive in nature

Compressive wave decelerates & expansion wave accelerates

Assumptions in studying Normal Shock

1) Shock wave is so thin that it can be considered to take place in constant Area.

2) Frictional effects are neglected. (very thin)

3) No external heat transfer ~~or work transfer~~ occurs since its very thin & can be approximated has adiabatic

4) No external work transfer

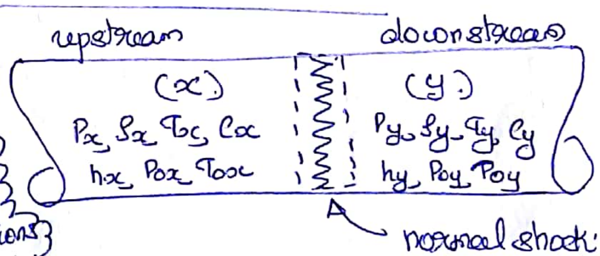
5) Body forces are negligible.

6) Flow is one-dimensional.

7) Perfect gas

Governing Equations in studying Normal Shock

repeat this figure for all derivations



1) Continuity Equation

$$\dot{m} = \dot{m}_x = \dot{m}_y$$

$$= \rho_x A C_x = \rho_y A C_y$$

$$\frac{\dot{m}}{A} = \rho_x C_x = \rho_y C_y = G$$

G = mass flux/mass velocity

2) Momentum Equation.

$$P_x A + \rho_x A C_x^2 = P_y A + \rho_y A C_y^2$$

$$P_x + \rho_x C_x^2 = P_y + \rho_y C_y^2.$$

3) Energy Equation $\quad h_o = h_{ox} + \frac{C_x^2}{2} = h_y + \frac{C_y^2}{2}.$

4) Equation of state $\quad h = f(P, \rho) \quad$ $P_x = \rho_x R T_x$

$\qquad s = f(P, \rho) \quad$ $P_y = \rho_y R T_y$

Mollier diagrams with Fanno line, Rayleigh line & normal shock. 6

Using equation (1), (3) & $h = f(s, s)$ we can draw Fanno line

Using equation (1), (2) & $s = f(s, P)$ we can draw Rayleigh line

[Figure 3 in page 6 of gas tables gives h-s diagram]

PRANDTL - MEYER RELATION

It gives the relation between velocities after and before shock and helps to obtain a relation between M_x & M_y.

For an isentropic process the steady flow energy equation reduces to the form

$$h + \frac{c^2}{2} \cancel{+ gz} = \text{Constant} \quad \left\{ h_1 + \frac{c_1^2}{2} = h_2 + \frac{c_2^2}{2} \right\}$$

Applying this equation at any point in isentropic flow and at the critical condition we can write:

$$h + \frac{c^2}{2} = h^* + \frac{c^{*2}}{2}$$

$$h = c_p T = \frac{\gamma R}{\gamma - 1} \times T = \frac{a^2}{\gamma - 1}$$

$$h^* = \frac{a^{*2}}{\gamma - 1}$$

$$a^* = c^*.$$

$$\frac{a^2}{\gamma - 1} + \frac{c^2}{2} = \frac{a^{*2}}{\gamma - 1} + \frac{a^{*2}}{2}.$$

$$\frac{a^2}{\gamma - 1} + \frac{c^2}{2} = \frac{a^{*2}}{2} \left\{ \frac{\gamma + 1}{\gamma - 1} \right\} \quad \text{— (1)}$$

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Applying Equation ⑦ before shock: (where flow ⑦ can be approx: $\Delta s = 0$)

$$\frac{a_x^2}{\gamma-1} + \frac{c_x^2}{2} = \frac{a^{*2}}{2}\left\{\frac{\gamma+1}{\gamma-1}\right\}$$

$$a_x^2 + \frac{c_x^2(\gamma-1)}{2} = \frac{a^{*2}}{2}(\gamma+1)$$

$$a_x^2 = \frac{\gamma+1}{2}a^{*2} - c_x^2\frac{\gamma-1}{2}$$

$$\frac{a_x^2}{c_x} = \frac{\gamma+1}{2}\frac{a^{*2}}{c_x} - c_x\frac{\gamma-1}{2} \quad — ②$$

$\because T_x^* = T_y^*$
$\Rightarrow a_x^* = a_y^*$

Applying Equation ⑦ after shock we get.

$$\frac{a_y^2}{c_x} = \frac{\gamma+1}{2}\frac{a^{*2}}{c_x} - \frac{\gamma-1}{2}c_x \quad — ③$$

Momentum Equation applied before & after shock we get.

$$P_xA + \rho_xAc_x^2 = P_yA + \rho_yAc_y^2$$

$$\dot{m}_x = \rho_xAc_x \qquad \dot{m}_y = \rho_yAc_y$$

$$P_xA + \dot{m}_xc_x = P_yA + \dot{m}_yc_y$$

$$P_xA - P_yA = \dot{m}_yc_y - \dot{m}_xc_x.$$

$$= \dot{m}(c_y - c_x) \quad \{\because \dot{m} = \dot{m}_x = \dot{m}_y\}$$

$$(P_x - P_y)\frac{A}{\dot{m}} = c_y - c_x.$$

From continuity Equation $\frac{\dot{m}}{A} = \rho_xc_x = \rho_yc_y$

$$(P_x - P_y) = \frac{\dot{m}}{A}c_y - \frac{\dot{m}}{A}c_x.$$

$$\frac{P_x - P_y}{\frac{\dot{m}}{A}} = c_y - c_x.$$

$$\frac{P_x}{\rho_x C_x} - \frac{P_y}{\rho_y C_y} = C_y - C_x \quad (3)$$

multiply entire equation by γ.

$$\frac{\gamma P_x}{\rho_x C_x} - \frac{\gamma P_y}{\rho_y C_y} = (C_y - C_x)\gamma \quad \text{---} (4)$$

From perfect gas equation $P_x = \rho_x R T_x$, $P_y = \rho_y R T_y$

$$\frac{P_x}{\rho_x} = R T_x \qquad \frac{P_y}{\rho_y} = R T_y$$

$$\frac{\gamma P_x}{\rho_x} = \gamma R T_x = a_x^2 \qquad \frac{\gamma P_y}{\rho_y} = \gamma R T_y = a_y^2$$

Substituting in equation (4)

$$\frac{a_x^2}{C_x} - \frac{a_y^2}{C_y} = \gamma (C_y - C_x) \quad \text{---} (5)$$

Subtracting equations (2) & (3) we get.

$$\frac{a_x^2}{C_x} - \frac{a_y^2}{C_y} = \left[\frac{\gamma+1}{2}\frac{a^{*2}}{C_x} - \frac{\gamma-1}{2} C_x\right] - \left[\frac{\gamma+1}{2}\frac{a^{*2}}{C_y} - \frac{\gamma-1}{2} C_y\right] \longrightarrow (6)$$

Equating equations (5) & (6) we get.

$$\gamma (C_y - C_x) = \frac{\gamma+1}{2}\frac{a^{*2}}{C_x} - \frac{\gamma-1}{2} C_x - \frac{\gamma+1}{2}\frac{a^{*2}}{C_y} + \frac{\gamma-1}{2} C_y$$

$$= \frac{\gamma+1}{2} a^{*2} \left\{\frac{1}{C_x} - \frac{1}{C_y}\right\} - \frac{\gamma-1}{2}\left\{C_x - C_y\right\}$$

$$\gamma (C_y - C_x) = \frac{\gamma+1}{2} a^{*2} \left\{\frac{C_y - C_x}{C_x C_y}\right\} + \frac{\gamma-1}{2}\left\{C_y - C_x\right\}$$

dividing entire equation by $(C_y - C_x)$ we get.

$$\gamma = \frac{\gamma+1}{2} a^{*2} \frac{1}{C_x C_y} + \frac{\gamma-1}{2}$$

Multiply entire equation by "$C_x C_y$"

$$\gamma C_x C_y = \frac{\gamma+1}{2} a^{*2} + \frac{\gamma-1}{2} C_x C_y$$

$$\gamma C_x C_y - \left(\frac{\gamma-1}{2}\right) C_x C_y = \frac{\gamma+1}{2} a^{*2}$$

$$C_x C_y \left\{\frac{2\gamma - \gamma + 1}{2}\right\} = \frac{\gamma+1}{2} a^{*2}$$

$$C_x C_y \left\{\frac{\gamma+1}{2}\right\} = \frac{\gamma+1}{2} a^{*2}$$

$$\boxed{C_x C_y = a^{*2}}$$

This Equation is known as PRANDTL-MEYER EQUATION

$$C_x C_y = a^* \times a^*$$

$$\frac{C_x}{a^*} \times \frac{C_y}{a^*} = 1.$$

$$a^* = \sqrt{\gamma R T_x^*} = \sqrt{\gamma R T_y^*}$$

$$\frac{C_x}{a_x^*} \times \frac{C_y}{a_y^*} = 1$$

$$\boxed{M_x^* \times M_y^* = 1}$$

M^* is the reference Mach number which ratio of fluid velocity at a point to sound velocity at point where $M = 1$.

$M_x^* \longrightarrow$ reference Mach number upstream of shock.

$M_y^* \longrightarrow$ reference Mach number downstream of shock.

RANKINE - HUGONIOT EQUATIONS

[For 12 marks start here]

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Relation / Equation connecting pressure ratio before & after shock with density ratio before and after shock.

PART I

Relation connecting M_x & M_y [Can be asked separately for 5 marks]

From SFEE applied to isentropic flow

$$h + \frac{c^2}{2} = \text{Constant}$$

Apply this equation at stagnation point & critical point (i.e. $M=1$)

$$h_0 = h^* + \frac{c^{*2}}{2} \quad \text{— I} \quad \{c_0 = 0\}$$

$$h^* = \frac{\gamma R T^*}{\gamma - 1} = \frac{a^{*2}}{\gamma - 1}$$

$$h_0 = \frac{\gamma R T_0}{\gamma - 1} = \frac{a_0^2}{\gamma - 1}$$

$$c^* = a^*$$

Therefore we can write I has

$$\frac{a_0^2}{\gamma - 1} = \frac{a^{*2}}{\gamma - 1} + \frac{c^{*2}}{2} = a^{*2}\left\{\frac{1}{\gamma - 1} + \frac{1}{2}\right\}$$

$$\frac{\gamma R T_0}{\cancel{\gamma - 1}} = \frac{a^{*2}}{2}\left\{\frac{\gamma + 1}{\cancel{\gamma - 1}}\right\}$$

$$a^{*2} = \frac{2\gamma R T_0}{\gamma + 1} \quad \text{—— (1)}$$

From Prandtl Meyer Relation we get

$$c_x c_y = a^{*2} \quad \text{—— (2)}$$

Substituting (2) in (1) we get

$$c_x c_y = \frac{2\gamma}{\gamma + 1} R T_0 \quad \text{—— (3)}$$

$$M_x = \frac{c_x}{a_x} \Rightarrow c_x = a_x M_x = M_x\sqrt{\gamma R T_x}$$ (11)

$$M_y = \frac{c_y}{a_y} \Rightarrow c_y = a_y M_y = M_y\sqrt{\gamma R T_y}$$

Substituting for c_x & c_y in eq (8)

$$M_x\sqrt{\gamma R T_x} \times M_y\sqrt{\gamma R T_y} = \frac{2\gamma}{\gamma+1} R T_0$$

$$M_x \times M_y \times \gamma R \times \sqrt{T_x} \times \sqrt{T_y} = \frac{2}{\gamma+1}\gamma R\, T_0$$

$$M_x \times M_y = \frac{2}{\gamma+1}\frac{T_0}{\sqrt{T_x T_y}}$$

Squaring the above equation

$$M_x^2 \times M_y^2 = \frac{4}{(\gamma+1)^2}\frac{T_0^2}{T_x T_y}$$

$$M_x^2 M_y^2 = \frac{4}{(\gamma+1)^2} \times \frac{T_0}{T_x} \times \frac{T_0}{T_y}$$

From Energy equation across shock $h_{ox} = h_{oy} = h_o$

$T_{ox} = T_{oy} = T_o$

$$M_x^2 M_y^2 = \frac{4}{(\gamma+1)^2} \times \frac{T_{ox}}{T_x} \times \frac{T_{oy}}{T_y}$$

The relation between static and stagnation condition is obtained through isentropic flow (since stagnation condition is achieved by isentropic deceleration)

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2 \quad \text{Page 4, Eq 3.2, gas tables}$$

For upstream of shock $\frac{T_{ox}}{T_x} = 1 + \frac{\gamma-1}{2}M_x^2$

For downstream of shock $\frac{T_{oy}}{T_x} = 1 + \frac{\gamma-1}{2}M_y^2$

$$M_x^2 \times M_y^2 = \frac{4}{(\gamma+1)^2} \times \left(1+\frac{\gamma-1}{2}M_x^2\right)\left(1+\frac{\gamma-1}{2}M_y^2\right)$$

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On rearranging & further simplification we get (after maths)

$$\boxed{M_y^2 = \frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1}M_x^2 - 1}}$$

→ Equation number 4.2 Page 5 Gas tables.

PART - 2 [Can be asked individually for 5 marks]

Static Pressure ratio across shock (P_y/P_x)

From Law of Conservation of momentum for Control volume.

$$\Sigma F = \frac{\partial M_{cv}}{\partial t} + \dot{M}_{out} - \dot{M}_{in}$$

For steady flow. $\Sigma F = \dot{M}_{out} - \dot{M}_{in}$

For very small thickness of the shockwave we can neglect body forces, viscous forces & turbulent forces leaving only pressure force.

Applying at before & after shock we get.

$$P_x A - P_y A = \dot{m}_x C_y - \dot{m}_x C_x$$

$$\cancel{P_y A - P_x A = \dot{m}_x C_y - \dot{m}_x C_x}$$

$$P_x A + \dot{m}_x C_x = P_y A + \dot{m}_y C_y$$

$$\dot{m}_x = \rho_x A C_x = \dot{m}_y = \rho_y A C_y$$

$$P_x A + \rho_x A C_x^2 = P_y A + \rho_y A C_y^2$$

$$P_x + \rho_x C_x^2 = P_y + \rho_y C_y^2$$

From perfect gas equation $P = \rho R T$

$P_x = \rho_x R T_x$ $\qquad$ $P_y = \rho_y R T_y$

$\gamma P_x = \gamma \rho_x R T_x$ $\qquad$ $\gamma P_y = \gamma \rho_y R T_y$

$\gamma P_x = \rho_x a_x^2$ $\qquad$ $\gamma P_y = \rho_y a_y^2$

$$a_x^2 = \frac{\gamma P_x}{\rho_x} \qquad a_y^2 = \frac{\gamma P_y}{\rho_y}$$

$$P_x + \rho_x C_x^2 = P_y + \rho_y C_y^2$$

$$P_x\left(1 + \frac{\rho_x C_x^2}{P_x}\right) = P_y\left(1 + \frac{\rho_y C_y^2}{P_y}\right)$$

$$P_x\left\{1 + \frac{\gamma \rho_x C_x^2}{\gamma P_x}\right\} = P_y\left\{1 + \frac{\gamma \rho_y}{\gamma P_y} C_y^2\right\}$$

we have $\frac{\gamma \rho_x}{P_x} = \frac{1}{a_x^2}$, $\frac{\gamma \rho_y}{P_y} = \frac{1}{a_y^2}$

Therefore the equation become.

$$P_x\left(1 + \gamma \frac{C_x^2}{a_x^2}\right) = P_y\left(1 + \gamma \frac{C_y^2}{a_y^2}\right)$$

$$P_x(1 + \gamma M_x^2) = P_y(1 + \gamma M_y^2)$$

$$\boxed{\frac{P_y}{P_x} = \frac{1 + \gamma M_x^2}{1 + \gamma M_y^2}}$$

Equation no 4.2
Page 5 from gas tables.

Substituting $M_y^2 = \dfrac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}$ in above equation (Eq no. 4.2)

$$\frac{P_y}{P_x} = \frac{1 + \gamma M_x^2}{1 + \gamma\left\{\dfrac{\frac{2}{\gamma-1} + M_x^2}{2\gamma/\gamma-1\, M_x^2 - 1}\right\}}$$

$$\frac{P_y}{P_x} = \frac{(1+\gamma M_x^2)\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right)}{\frac{2\gamma}{\gamma-1} M_x^2 - 1 + \frac{2\gamma}{\gamma-1} + \gamma M_x^2}$$

On rearranging and further simplification we get.

$$\boxed{\frac{P_y}{P_x} = \frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1}}$$

Equation no. 4.3
Page 5, gas tables.

PART - 2

[Can be asked separately for 5 marks]

Static temperature ratio across shock (T_y/T_x)

The flow before and after shock can be considered isentropic.

From isentropic relations $\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$

Page no. 4
Eq 3.2.
gas tables.

Before shock $\frac{T_{0x}}{T_x} = 1 + \frac{\gamma-1}{2} M_x^2$

After shock $\frac{T_{0y}}{T_{y}} = 1 + \frac{\gamma-1}{2} M_y^2$

$$\frac{T_y}{T_x} = \frac{T_y/T_{0y}}{T_x/T_{0x}} \qquad \left\{ \because T_{0x} = T_{0y} \right\}$$

$$\boxed{\frac{T_y}{T_x} = \frac{1 + \frac{\gamma-1}{2} M_x^2}{1 + \frac{\gamma-1}{2} M_y^2}}$$

Equation 4.4
Page 5
Gas tables

Substituting $M_y^2 = \dfrac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}$ (18) Eq 4-2, Page 5, gas tables

$$\frac{T_y}{T_x} = \frac{2 + (\gamma-1) M_x^2}{2 + (\gamma-1)\left\{\dfrac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma M_x^2}{\gamma-1} - 1}\right\}}$$

$$\frac{T_y}{T_x} = \frac{(2 + (\gamma-1) M_x^2)(2\gamma M_x^2 - \gamma + 1)}{2(2\gamma M_x^2 - \gamma + 1) + (\gamma-1)(2 + M_x^2(\gamma-1))}$$

Rearranging & simplifying we get

$$\boxed{\frac{T_y}{T_x} = \frac{\left(1 + \frac{\gamma-1}{2} M_x^2\right)\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right)}{\frac{(\gamma+1)^2 M_x^2}{2(\gamma-1)}}}$$

Equation 4.4 Page 5. Gas tables

PART 4

density ratio across shock { For 5 marks start rankine Hugonoit equation from here }

From perfect gas equation

$$\frac{P}{\rho T} = \text{Constant } (R)$$

Applying this before & after shock we get

$$\frac{P_x}{\rho_x T_x} = \frac{P_y}{\rho_y T_y}$$

$$\frac{\rho_y}{\rho_x} = \frac{P_y}{P_x} \times \frac{T_x}{T_y} = \frac{\left(\frac{P_y}{P_x}\right)}{(T_y/T_x)}$$

Substitute $\frac{P_y}{P_x}$ & $\frac{T_y}{T_x}$ interms of M_x. {Equation 4.3 & 4.4, Page 5, G.table}

$$\frac{\rho_y}{\rho_x} = \left\{\frac{2\gamma}{\gamma-1} M_x^2 - \frac{\gamma-1}{\gamma+1}\right\} \left\{\frac{\frac{(\gamma+1)^2}{2(\gamma-1)} M_x^2}{\left(1+\frac{\gamma-1}{2} M_x^2\right)\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right)}\right\}$$

rearranging & simplifying we get.

$$\boxed{\frac{\rho_y}{\rho_x} = \frac{\left(\frac{\gamma+1}{2}\right) M_x^2}{1+\frac{\gamma-1}{2} M_x^2}}$$

By C ——— (I)

from $\frac{P_y}{P_x} = \frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1}$

obtain M_x. $\Rightarrow M_x^2 = \frac{\gamma+1}{2\gamma} \frac{P_y}{P_x} + \frac{\gamma-1}{2\gamma}$.

$$\frac{\rho_y}{\rho_x} = \frac{P_y}{P_x} \times \frac{T_x}{T_y}$$

Substitute T_x/T_y in term of M_x in above equation

$$\frac{\rho_y}{\rho_x} = \frac{P_y}{P_x} \times \frac{\frac{(\gamma+1)^2 M_x^2}{2(\gamma-1)}}{\left(1+\frac{\gamma-1}{2} M_x^2\right)\left(\frac{2\gamma}{\gamma-1} M_x^2 - 1\right)}$$

Substitute for M_x in terms of $\frac{P_y}{P_x}$.

$$\frac{\rho_y}{\rho_x} = \frac{P_y}{P_x} \times \frac{\frac{(\gamma+1)^2}{2(\gamma-1)}\left\{\frac{\gamma+1}{2\gamma}\frac{P_y}{P_x} + \frac{\gamma-1}{2\gamma}\right\}}{\left[1 + \frac{\gamma-1}{2}\left\{\frac{\gamma+1}{2\gamma}\frac{P_y}{P_x} + \frac{\gamma-1}{2\gamma}\right\}\right]\left[\frac{2\gamma}{\gamma-1}\left\{\frac{\gamma+1}{2\gamma}\frac{P_y}{P_x} + \frac{\gamma-1}{2\gamma}\right\} - 1\right]}$$

Rearranging & simplification we get

$$\boxed{\frac{\rho_y}{\rho_x} = \frac{1 + \left(\frac{\gamma+1}{\gamma-1}\right)\frac{P_y}{P_x}}{\frac{\gamma+1}{\gamma-1} + \frac{P_y}{P_x}}} \qquad \text{— II}$$

By ♡

On rearranging we can also write

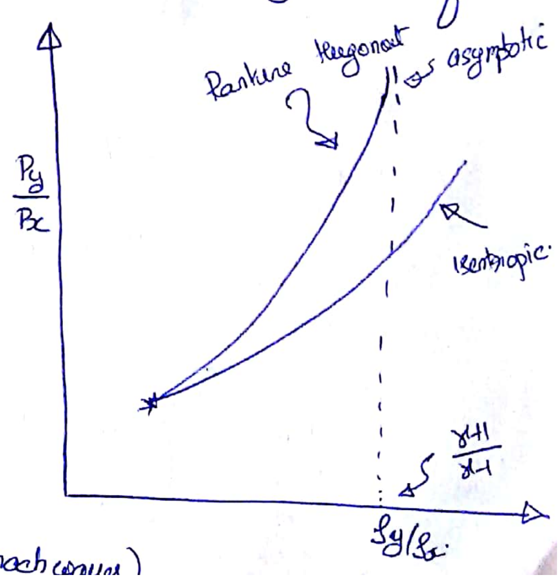
$$\boxed{\frac{P_y}{P_x} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right)\frac{\rho_y}{\rho_x} - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_y}{\rho_x}}} \qquad \text{— III}$$

By ♡

Equation II & III are called Rankine Hugoniot Equation

For shock P_y/P_x = [above equation]

For isentropic process for same pressure ratio $\frac{P_y}{P_x} \cdot \frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$ [$pv^{\gamma} = \text{Const}$]



For lower value of P_y/P_x i.e.,.... weak shock wave Hugoniot curve approaches isentropic curve. implies weak shocks can be studied as isentropic (mach waves)

<u>Change in entropy across shock.</u> 18

change in entropy ~~across~~ in a process is given by

$$\Delta S = S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$ { 3.5_thermo By ♡ }

Applying this equation across shock we get:

$$\Delta S = S_y - S_x = C_p \ln \frac{T_y}{T_x} - R \ln \frac{P_y}{P_x}$$

$$\Delta S = C_p \ln \frac{T_y}{T_x} - C_p\left(\frac{\gamma-1}{\gamma}\right) \ln \frac{P_y}{P_x}$$ $\left\{ C_p = \frac{\gamma R}{\gamma-1},\ R = C_p\left(\frac{\gamma-1}{\gamma}\right) \right\}$

{ Across shock we can approximate flow to be adiabatic. ($pv^{\gamma} = c$)

$$\therefore \frac{P_y}{P_x} = \left(\frac{T_y}{T_x}\right)^{\frac{\gamma}{\gamma-1}}$$ }

$$\Delta S = C_p \ln \frac{T_y}{T_x} - C_p\left(\frac{\gamma-1}{\gamma}\right) \cancel{\ln\left(\frac{T_y}{T_x}\right)^{\frac{\gamma}{\gamma-1}}} \times \ln \frac{P_y}{P_x}$$

$$\Delta S = C_p \left\{ \ln \frac{T_y}{T_x} - \ln\left(\frac{P_y}{P_x}\right)^{\frac{\gamma-1}{\gamma}} \right\}$$

$$\Delta S = C_p \frac{\ln (T_y/T_x)}{\ln (P_y/P_x)^{\frac{\gamma-1}{\gamma}}} \quad \text{--- (1)}$$

$$\frac{T_y}{T_x} = \frac{T_y/T_{oy}}{T_x/T_{ox}} = \frac{\left(1+\frac{\gamma-1}{2} M_x^2\right) T_{oy}}{\left(1+\frac{\gamma-1}{2} M_y^2\right) T_{ox}} \quad \text{--- (2)}$$

$$\frac{P_y}{P_x} = \left(\frac{T_y}{T_x}\right)^{\frac{\gamma}{\gamma-1}}$$

(19)

$$\left(\frac{P_y}{P_x}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{T_y}{T_x}\right)^{\frac{\gamma}{\gamma-1}\times\frac{\gamma-1}{\gamma}} = \frac{T_y}{T_x}.$$

$$\left(\frac{P_y}{P_x}\right)^{\frac{\gamma-1}{\gamma}} = \frac{1+\frac{\gamma-1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_y^2} \times \frac{T_{oy}}{T_{ox}}.$$

$$\frac{P_{oy}}{P_{ox}} = \left(\frac{T_{oy}}{T_{ox}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow \left(\frac{P_y}{P_x}\right)^{\frac{\gamma-1}{\gamma}} = \frac{1+\frac{\gamma-1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_y^2} \times \left(\frac{P_{oy}}{P_{ox}}\right)^{\frac{\gamma-1}{\gamma}}. \quad \text{——— (3)}$$

Substitute (2) & (3) in (4)

Rearranging (1) $\Rightarrow \Delta S = C_p \ln\left[\frac{\frac{T_y}{T_x}}{(P_y/P_x)^{\frac{\gamma-1}{\gamma}}}\right]$ ——— (4)

$$\Delta S = C_p \ln\left[\frac{\frac{T_{oy}}{T_{ox}}}{\left(\frac{P_{oy}}{P_{ox}}\right)^{\frac{\gamma-1}{\gamma}}}\right]$$

$$T_{ox} = T_{oy}$$

$$\Delta S = C_p \ln\left(\frac{P_{oy}}{P_{ox}}\right)^{-\frac{\gamma-1}{\gamma}}$$

$$= + C_p \frac{\gamma-1}{\gamma} \ln\left(\frac{P_{oy}}{P_{ox}}\right)^{-1}$$

$$\Delta S = R \ln\left(\frac{P_{ox}}{P_{oy}}\right) \quad \text{——— (5)}$$

$$\frac{P_{ox}}{P_{oy}} = \frac{P_y P_x P_{ox}}{P_{oy} P_y P_x}$$

$$\frac{P_{ox}}{P_x} = \left[1+\frac{\gamma-1}{2} M_x^2\right]^{\frac{\gamma}{\gamma-1}} , \quad \frac{P_y}{P_{oy}} = \left[1+\frac{\gamma-1}{2} M_y^2\right]^{\frac{\gamma}{1-\gamma}}$$

$$\frac{P_x}{P_y} = \frac{1}{\frac{2\gamma M_x^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1}}$$

$$M_y^2 = \frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}$$

$$\frac{P_{ox}}{P_{oy}} = \left[1+\frac{\gamma-1}{2} M_x^2\right]^{\frac{\gamma}{1-\gamma}} \times \left[\frac{1}{\frac{2\gamma M_x^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1}}\right] \times \left[1+\frac{\gamma-1}{2} M_x^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{ox}}{P_{oy}} = \left[1+\frac{\gamma-1}{2}\left\{\frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}\right\}\right]^{\frac{\gamma}{1-\gamma}} \left[\frac{2\gamma M_x^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1}\right]^{-1} \times \left[1+\frac{\gamma-1}{2} M_x^2\right]^{\frac{\gamma}{\gamma-1}}$$

Rearranging & simplification

$$\frac{P_{ox}}{P_{oy}} = \left[\frac{\frac{\gamma+1}{2} M_x^2}{1+\frac{\gamma-1}{2} M_x^2}\right]^{\frac{\gamma}{1-\gamma}} \left[\frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1}\right]^{\frac{1}{\gamma-1}} \quad (6)$$

By ♡

Substitute (6) in (5).

$$\Delta S = R \ln\left\{ \left[\frac{\frac{\gamma+1}{2} M_x^2}{1 + \frac{\gamma-1}{2} M_x^2} \right]^{\frac{\gamma}{1-\gamma}} \times \left[\frac{2\gamma M_x^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1} \right]^{\frac{1}{\gamma-1}} \right\}$$

$$\Delta S = R \ln\left[\frac{\frac{\gamma+1}{2} M_x^2}{1 + \frac{\gamma-1}{2} M_x^2} \right]^{\frac{\gamma}{1-\gamma}} + R \ln\left[\frac{2\gamma M_x^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1} \right]^{\frac{1}{\gamma-1}}$$

$$\Delta S = R \frac{\gamma}{1-\gamma} \ln\left[\frac{1 + \frac{\gamma-1}{2} M_x^2}{\frac{\gamma+1}{2} M_x^2} \right]^{-1} + R \frac{1}{\gamma-1} \ln\left[\frac{2\gamma M_x^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1} \right]$$

$$\boxed{\Delta S = R \times \frac{\gamma}{\gamma-1} \ln\left[\frac{2}{(\gamma+1) M_x^2} + \frac{\gamma-1}{\gamma+1} \right] + R \frac{1}{\gamma-1} \ln\left[\frac{2\gamma M_x^2}{\gamma+1} - \frac{\gamma-1}{\gamma+1} \right]}$$

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Can Shock occur in Subsonic & Supersonic Flow?

The change in entropy across shock is given by

$$\Delta S = R\left\{ \frac{\gamma}{\gamma-1} \ln\left\{ \frac{2}{(\gamma+1)M_x^2} + \frac{\gamma-1}{\gamma+1} \right\} + \frac{1}{\gamma-1} \ln\left\{ \frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1} \right\} \right\}$$

For $M_x < 1$ (Subsonic flow) violation of 2nd law hence not possible.

ΔS = negative.

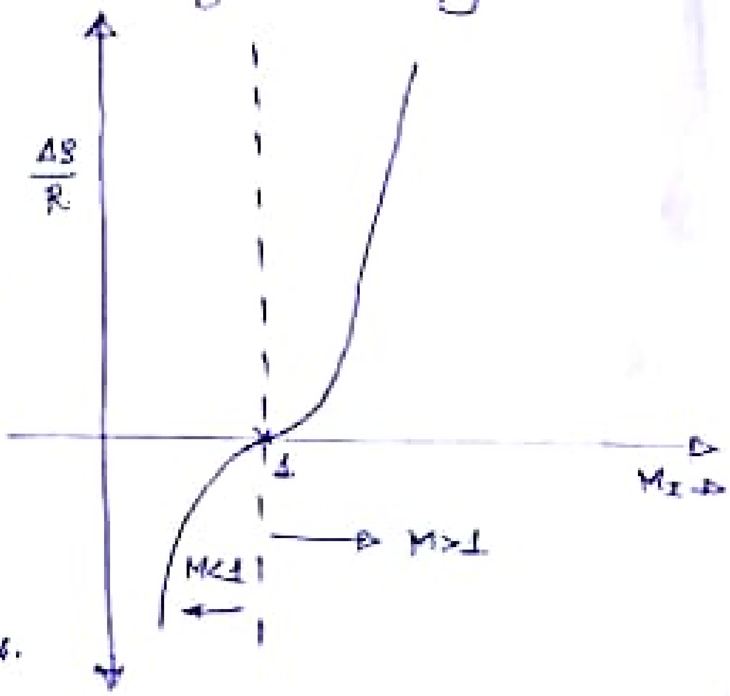
For $M_x > 1$. (Supersonic flow)

ΔS = positive

For $M_x = 1$. (sonic flow.

$\Delta S = 0$

A plot of ΔS versus M_x for given value of γ & R.



Also draw figure, figure 8, Page 6, gas tables.

going from $x \rightarrow y$ h-s diagram gives ΔS positive

If it was reverse i.e. $y \rightarrow x$ ΔS will be negative which is violation of 2nd law of thermodynamics.

Strength of a shock wave.

is defined as the ratio of difference in static pressure across shock to the upstream pressure.

$$\beta = \frac{P_y - P_x}{P_x} = \frac{P_y}{P_x} - 1$$

We can write P_y/P_x in terms of M_x. [Eq 4.3 Page 5]

$$\beta = \frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1} - 1$$

$$\beta = \frac{2\gamma}{\gamma+1} M_x^2 - \frac{(\gamma-1) - (\gamma+1)}{\gamma+1}$$

$$\beta = \frac{2\gamma M_x^2 - 2\gamma}{\gamma+1}$$

$$\beta = \frac{2\gamma}{\gamma+1} (M_x^2 - 1)$$

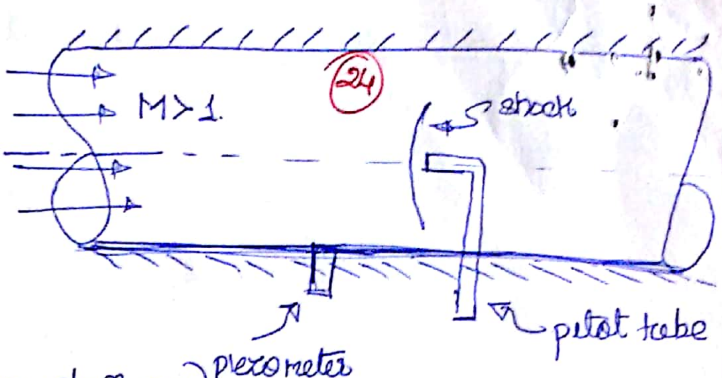
$$\beta \propto (M_x^2 - 1)$$

* Greater the value of M_x (upstream mach no.), greater will be the strength of shock.
* Lower value of M_x, lower will be the strength. also referred has waves of ~~a~~ vanishing strength.

Determination of Mach number of a supersonic flow.

Consider a duct of constant area with fluid flowing at supersonic speed. A small obstruction in its path can ~~cause~~ result in shock wave in the flow. This is utilized to determine the flow Mach number ~~the~~ ~~analysis~~ by analysing the shock formed.

For do the analysis, a pitot tube is introduced in the supersonic flow. This will act as an obstruction resulting formation of shock in front of pitot tube. Thus pitot tube will measure stagnation pressure after shock (Pitot tubes used for measuring stagn press)



Placing a piezometer upstream of pitot tubes position (such that it comes before shock) we can measure static pressure before shock.

Hence from this step-up we can obtain P_x & P_{oy}

For analysis we will use normal shock equations

$$\frac{P_{oy}}{P_x} = \frac{P_{oy}}{P_y} \times \frac{P_y}{P_x} \quad \{\text{Equation 4.11, Page 5, Gas tables}\}$$

$\frac{P_{oy}}{P_y}$ refers to ~~isentropic~~ pressure ratio (stagnant to static) & can be obtained from isentropic relations.

$$\frac{P_{oy}}{P_y} = \left[1 + \frac{\gamma-1}{2} M_y^2\right]^{\frac{\gamma}{\gamma-1}}$$

Substituting for M_y^2 using {Eq 4.2, Page 5}

$$\frac{P_{oy}}{P_y} = \left\{ 1 + \frac{\gamma-1}{2} \left\{ \frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1} \right\} \right\}^{\frac{\gamma}{\gamma-1}}$$

$$P_y/P_x = \frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1} \quad \{\text{Eq 4.3, Page 5}\}$$

Substituting

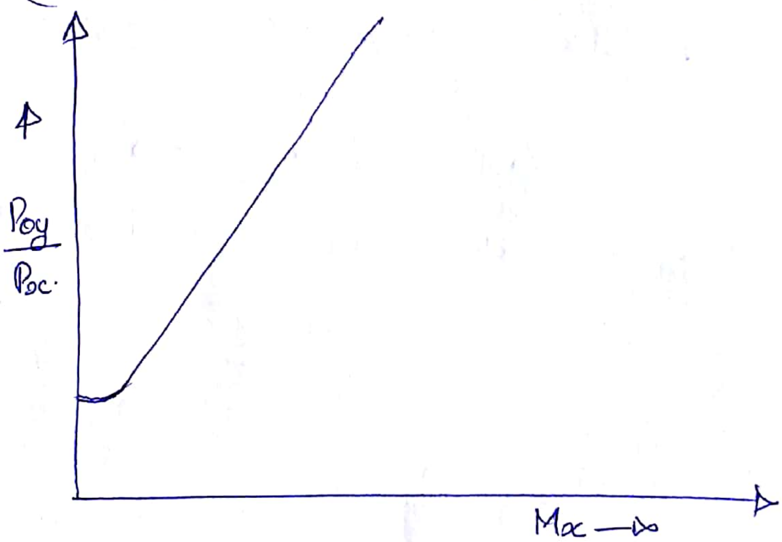
$$\frac{P_{oy}}{P_x} = \left\{ \frac{\frac{4\gamma}{\gamma-1} M_x^2 - 2 + 1 + \frac{M_x^2(\gamma-1)}{2}}{\frac{2\gamma}{\gamma-1} M_x^2 - 1} \right\} \left\{ \frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1} \right\}$$

$$\therefore \frac{P_{oy}}{P_x} = \left\{ \frac{\dfrac{4\gamma M_x^2 - (\gamma+1) + \frac{M_x^2(\gamma-1)^2}{2}}{(\gamma+1)}}{\dfrac{2\gamma}{\gamma-1} M_x^2 - 1} \right\} \left\{ \frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1} \right\}$$

rearranging & simplification

$$\frac{P_{oy}}{P_x} = \left[\frac{\left(\frac{\gamma+1}{2} M_x \right)^{\frac{\gamma}{\gamma-1}}}{\left(\frac{2\gamma}{\gamma+1} M_x^2 - \frac{\gamma-1}{\gamma+1} \right)^{\left(\frac{1}{\gamma-1}\right)}} \right]$$

A plot between (P_{oy}/P_x) and M_x gives the following



By obtaining the above plot we can easily obtain M_x value for any given value of $\frac{P_{oy}}{P_x}$

~

Things to Remember for doing Numerical Questions

1) $\rho_x C_x = \rho_y C_y$ { mass flow rate remains constant }

2) $T_{0x} = T_{0y}$

$h_{0x} = h_{0y}$

3) $P_x = \rho_x R T_x$

$P_y = \rho_y R T_y$

4) Unless otherwise specified flow upstream & downstream to be considered as isentropic.

5) Relation between static & stagnation condition will be available in isentropic tables.

6) Obtain M_x or M_y. Getting one value will lead to the other from shock tables.